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On Testing the Statistical Significance of Indices of Poverty : Some Results

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Summary

Some asymptotic tests of significance regarding two well known measures of poverty viz poverty gap ratio and Takayama's censored Gini ratio have been obtained. The sampling distribution of Takayama's censored Gini ratio is also obtained. An empirical illustration on the use of the testing procedure is provided.

Key words : Inequality; Poverty; Gini-ratio; Censored Distribution.

Introduction

"No society can surely be flourishing and happy of which by far the greater part of the numbers are poor and miserable", observed Adam Smith (1976), The Father of Modern Economics. But today in spite of some two centuries of global progress, more than three- fourths of the humanity stands inadequately provided with the means of existence. These less fortunates, called the poor, who are concentrated in the Third World, have attracted considerable attention of the academic world and policy matters alike (see Todaro [16]). Consequently, studies on poverty outnumber any other area in the field of Economic development. Almost all policies on the Third World are examined vis-a-vis incidence of poverty.

Though, both, conceptualization and measurement of poverty is considered a tricky domain, no policy evaluation can do without reference to some or the other index chosen for the purpose. In the eighties, the Economic and Political Weekly (various issues) published several articles touching these problems, (see Julka [8], Arora *et al.* [1]). Starting with the conventional Head - count ratio, currently we have some twenty indices of poverty touching one or more aspects of absolute deprivation (see Sen [13], Julka [8], [10], [7], [[9]). Thus in policy evaluation exercises, it is customary to compute one or more indices of poverty and to compare their values to assess the efficacy of poverty alleviation programmes. Both convenience and cost considerations invariably permit the use of sample data for empirical estimation. Under the circumstances those evaluation statements lose much of their charm for want of tests of significance.

Accordingly, this paper is an attempt at developing the tests of significance for some basic indices of poverty. We have confined ourselves to the following two measures :

(i) Poverty – Gap ratio; and

(ii) Takayama's Censored Gini-ratio.

Since poverty is primarily viewed as deprivation it is the nation of poverty-gap that constitutes the essence of poverty measurement. Therefore, the present exercise can provide a clue to many other indices besides the ones included here.

To demonstrate the workability of the test-statistics, developed in this paper, an empirical illustration is provided in Section 3.

Preliminaries

On the face of it, measurement of poverty involves just two neat steps viz., identification and aggregation. But the serious students of this complex phenomenon are well aware of the numerous difficulties underlying these procedures. This simple looking term (poverty) has come in for some many passionate descriptions, axiological prescriptions and mechanistic subscriptions that a mystical dimension seems to have engulfed the naked truth. It is beyond the scope of this paper to go into all those aspects. Therefore, assuming those problems away, we start with a given distribution profile $y_1, y_2, \ldots, y_n$ for n units drawn from a population F(Y). Let Z be the chosen poverty norm so that the units are designated as poor or non-poor according to the condition :

• Sen's index [12] can be interpreted as weighted poverty gap index for the poor, as it is given by

$$P = \frac{2}{(p+1)nz} \sum_{i=1}^{p} g_i\,(p+1-i)$$

where Z denotes poverty norm, p denotes number of poor, g_i = poverty gap for the ith person and (p+l–i) = weights for the ith person.

Similarly, Thon's [15] index of poverty is also the normalised weighted sum of poverty gap of the poor.

It may be mentioned in passing that one of the simplest yet most famous measures of poverty is "Head Count Ratio". Since the measure admits directly the usual "test of proportion", the details are omitted here.

$$y_i \leq Z \quad i \in P$$

$$y_i > Z \quad i \in NP \tag{1}$$

where P stands for the set of the Poor and NP that of the non- poor.

Let p out of these n units be poor. Arrange y_i 's in ascending order such that

$$y_1 \leq y_2 \leq \ldots \leq y_p \leq y_{p+1} \leq \ldots \leq y_n \tag{2}$$

The income gap (or poverty gap) of the ith unit is

$$g_i = (Z - y_i) \tag{3}$$

The total poverty gap for the poor is

$$g = \sum_{i=1}^{p} g_i = \sum_{i=1}^{p} (Z - y_i) \tag{4}$$

and the average poverty gap :

$$g^* = \frac{g}{p} \tag{5}$$

Now, we define below both the indices of poverty and the associated tests of significance.

1. Poverty–Gap Ratio

An important index of poverty taken up by us is the "poverty-gap ratio" or "income-gap ratio", which is defined in two alternative ways, a la' Sen [13].

$$I_1 = \frac{g^*}{Z} \quad 1 - \frac{\bar{y}}{Z} \tag{6}$$

and $$I_2 = \frac{g^*}{\mu} \tag{7}$$

where, $\bar{y}$ = mean income of the poor
μ = mean income of the whole population.

The measure reflects the intensity of poverty suffered by poor and the inequality of income among the poor. For developing the test of significance, we have concentrated on the first version only viz I_1. The following section deals with the tests of significance for comparing two estimates of poverty; based on two independent samples of income derived from some unknown income distributions.

1.a Testing the significance for Difference Between Poverty Gap Ratios Based on Two Independent Samples.

Let n_1 be the sample of observations from an income distribution with $I_1^{(1)}$ as the poverty gap ratio. Also, let $I_1^{(2)}$ be the value of poverty gap ratio based on another random sample of size n_2 from another income distribution. Further, assume that both the samples are drawn independently of each other.

Suppose

p_1 = number of poor in the first sample

p_2 = number of poor in the second sample

Then testing the hypothesis

$$H_O : I_1^{(1)} = I_1^{(2)} \tag{8}$$

is equivalent to testing $$\frac{g_1^*}{Z} = \frac{g_2^*}{Z} \tag{9}$$

or $$g_1^* = g_2^* \tag{10}$$

or $$\sum_{i=1}^{p_1} \frac{(z-y_i)}{p_1} = \sum_{i=1}^{p_2} \frac{(z-y_i)}{p_2} \tag{11}$$

or $$z - \sum_{i=1}^{p_1} \frac{y_i}{p_1} = z - \sum_{i=1}^{p_2} \frac{y_i}{p_2} \tag{12}$$

or Mean income of the poor in 1st sample = Mean income of the poor in 2nd sample (13)

Since the population is unknown, and we have knowledge of both p_1 and p_2, and the samples consisting of both p_1 and p_2 are independent of each other, the classical nonparametric "Mann-Whitney U-statistics" can be applied to test the difference in location of two samples based on p_1 and p_2 number of observations. (See Gibbons [5]). In brief, the following testing procedure can be applied :

Assuming that p_1 corresponds to the smaller sample and p_2 to be larger sample, the test statistic is given by

$$U = p_1 p_2 + \frac{p_1 (p_1 + 1)}{2} - R_1 \tag{14}$$

where R_1 denotes the ranks of observations from samples of size p_1, when ranking is done for sample of size $(p_1 + p_2) = p$ (say).

When sample is large, i.e. if p is large, the appropriate test - statistic is given by (under H_0)

$$Z = \frac{U - E(U)}{\sqrt{V(U)}} \sim N(0, 1) \tag{15}$$

where, $E(U) = \frac{P_1 P_2}{2}$ and $V(U) = \frac{P_1 P_2 (p + 1)}{12}$

We reject H_0 in favour of one-sided or two-sided alternative at $\alpha\%$ or $\alpha/2\%$ level of significance.

2. Takayama's Censored Gini Ratio

Proposed by Takayama [14], the index is based on the idea of "Censored distribution" as explored by Hamda & Takayama [6]. The index is the translation of usual Gini ratio of inequality to the "Censored income distribution" called Takayama's poverty index. The censored income distribution is obtained from the actual distribution by replacing all incomes above poverty line by incomes exactly equal to the poverty line i.e.

$$y_i = Z \ \forall \ i > p \qquad \text{where} \quad i = 1, \ldots, n.$$

The "Gini index" of this censored distribution is known as "Takayama's censored Gini ratio", defined as

$$\hat{T} = \frac{\frac{1}{n(n-1)} \sum_{i}^{n} \sum_{j}^{n} | y_i^* - y_j^* |}{2 \sum_{i=1}^{n} \frac{y_i^*}{n}} \tag{16}$$

where

$$y_i^* = y_i \qquad \text{if} \quad y_i \leq z$$
$$= z \qquad \text{otherwise}$$

Considering $y_1^*, \ldots, y_n^*$ as the random sample of size n from the censored income distribution, the sample estimate of "Takayama's censored Gini ratio" is obtained on the same lines as that of the sample estimate of 'Gini index' of inequality. This resemblance between the two has facilitated the task to obtain the sampling distribution of $\hat{T}$ and the associated test of significance.

2.a Sampling Distribution of Takayama's Censored Gini Ratio

The sampling distribution of $\hat{G}$ has already been developed by us (See. Arora et. al. [2]). Moving on the same lines the sampling distribution of $\hat{T}$ will be obtained in this paper.

Let the incomes in the censored population be ordered as

$$y_1^* \leq y_2^* \leq \ldots \leq y_n^*$$

Then the equivalent definition of $\hat{T}$ can be written as (compare Arora et. al. [2], pp 120).

$$\hat{T} = \frac{\hat{\Delta}}{2\mu} \tag{17}$$

where $\hat{\Delta}$ = Mean difference in the sample with observations $y_1^*, y_2^*, \ldots, y_n^*$

and $\hat{\mu}$ = sample mean for y_i^*, $i = 1, \ldots, n$.

Now, considering $y_1^*, y_2^*, \ldots, y_n^*$ as a random sample of size n, the sampling distribution of T can be obtained in the same way as that of $\hat{G}$ (Ref. Ramakrishna [11] and Arora *et. al.* [2].

Theorem : Asymptotic distribution $\{n(\hat{T}-T)\}^{1/2}$ is $N(0, \sigma_t^2)$

where

$$\sigma_t^2 = \frac{\sigma_{11}}{4\mu^2} + T^2\frac{\sigma_{22}}{\mu^2} - T\frac{\sigma_{12}}{\mu^2}$$

$$\sigma_{11} = \lim_{n\to\infty} n \operatorname{var}(\hat{\Delta}) = 4 v [E |y^{**} - y_2^*|]$$

and

$V[E \mid y^{**} - y_2^* \mid]$ = variance of the conditional expectation of $| y^{**} - y_2^* |$, given y_1^* is equal to some fixed value, say, y^{**},

$$\sigma_{12} = \lim_{n\to\infty} n \operatorname{Cov}(\hat{\Delta}, \bar{y}^*) = 2 \operatorname{Cov}(E \mid y^{**} - y_2^* \mid, y^{**})$$

$$\sigma_{22} = \lim_{n\to\infty} n \operatorname{var}(\bar{y}^*) = \sigma^2$$

$T = \dfrac{\Delta}{2\mu}$ = Censored Gini ratio in the population.

where, further, Δ = the population mean difference

μ = population mean.

Remark : In the above result, σ_t^2 is estimated by

$$\hat{\sigma}_t^2 = \frac{1}{(\bar{y}^*)^2}[\operatorname{var}\hat{\Delta}_i - 2\hat{T}\operatorname{Cov}(T_i, y_i^*) + \hat{T}^2 \operatorname{Var}(y_i^*)]$$

where,

$$\operatorname{Var}(\hat{\Delta}_i) = \frac{1}{n}\sum_{i=1}^{n} \hat{\Delta}_i^2 - (\hat{\Delta})^2$$

$$\hat{\Delta}_i = \left[\frac{(2i-n-1)\,y_i^* + ti}{n-1}\right]$$

and

$$t_i = \sum_{j>i}^{n} y_j^* - \sum_{j<i}^{n} y_j^* \quad ; \quad \hat{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \hat{\Delta}_i$$

$$\text{Cov}(\hat{\Delta}_i, y_i^*) = \frac{1}{n}\sum_{i=1}^{n} \hat{\Delta}_i y_i^* - \bar{y}^* \; ; \qquad \bar{y}^* = \frac{1}{n}\sum_{i=1}^{n} y_i^*$$

$$\text{Var}(y_i^*) = \frac{1}{n}\sum_{i=1}^{n} (y_i^* - \bar{y}^*)^2$$

Proof: Both the above results follow directly by putting $y_i = y_i^*$ and $y^* = y^{**}$ in the results of Lemma and Remark by Arora et. al. ([2], 124-125).

2.b. Tests of Significance for Takayama's Censored Gini ratio

The tests of significance follow exactly the same lines as that of $\hat{G}$, for large n. For illustrative purposes let us take the case of two independent samples.

Let $\hat{T}_1$ and $\hat{T}_2$ be two estimates of Modified Takayama's Censored Gini ratio based on two independent samples of sizes n_1 and n_2 respectively.

Then under $H_O: T_1 = T_2$ the appropriate test statistic is given by

$$Z = \frac{\hat{T}_1 - \hat{T}_2}{\left(\frac{\hat{\sigma}_{t_1}^2}{n_1} + \frac{\hat{\sigma}_{t_2}^2}{n_2}\right)^{1/2}} \sim N(0, 1) \qquad \text{for large n,}$$

where T_1, T_2 is the population value of $\hat{T}_1$ and $\hat{T}_2$ respectively and

$\frac{\hat{\sigma}_{t_1}^2}{n_1}$ = asymptotic variance of T_1

$\frac{\hat{\sigma}_{t_2}^2}{n_2}$ = asymptotic variance of T_2.

3. Illustrative Example

Since this is merely an illustrative example, aimed at demonstrating the working of the test and computational procedure; we have relied on the primary data collected by Bagai & Soni [3] in their ICAR project. The poverty norm used in this exercise has been taken from Julka [8].

Farm Business Income profiles A and B of 156 households were obtained using the raw data flowing from the above cited project. While profile A represents the existing income distribution in the sampled population, the profile B has been generated through a simulation exercise visualising an allocationally efficient farm economy on the lines of Chopra [4]. These income profiles were found to be statistically independent.

To examine the null hypothesis of no significant difference in the incidence of poverty (as measured through poverty gap ratio and Takayama's censored Gini ratio) amongst the farming households, the test-statistics developed in the paper can be easily applied. For the purpose of this exercise a poverty norm of Rs. 698/- (per capita per annum) at 1979-80 prices is used.

Computed values of the various entities required for "Poverty gap ratio" come out to be as under :—

	Poverty gap ratio (I_1)	p (number of poor)	U – statistic	Z – statistic
Profile A	0.8979	102	1992	– 3.2863*
Profile B	0.6394	57		

*Significant at 5% level

From the above result it is concluded that there is a significant difference in the incidence of poverty in two situations and further a significantly higher incidence of poverty is prevalent in profile A as compared to profile B.

For Takayama's censored Gini ratio, the relevant values are as under :

	$\hat{T}$	Asy Var of $\hat{T}$	Z – statistic
Profile A	0.7763	0.0148	4.325449*
profile B	0.2326	0.0010	

*Significant at 5% level

The results do affirm the earlier drawn inferences, on the basis of poverty gap ratio that there is significant difference in the incidence of poverty in the two profiles. Further, we find that profile A has a significantly higher incidence of poverty than the one prevalent in profile B.

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